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# AMERICAN UNIVERSITY OF BEIRUT
# FACULTY OF ENGINEERING AND ARCHITECTURE
## MECH 230 - DYNAMICS – QUIZ 2 – SPRING 2013

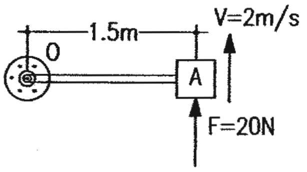
**NAME:** **ID No:** **Apr. 22, 2013**

**1.5 hours closed book quiz**

- Solve on the answer booklet in sequence.
- Question booklet will not be corrected and must be returned.
- Write clearly. Clarity is important in grading.
- Vectors are indicated in bold.
- Take g=9.81 m/s$^2$.

No.1 – (25%)

At t=0 s the 5 kg block, attached to the rigid member OA, is rotating with a speed of 2 m/s around the circular path centered at O on the horizontal plane when a constant tangential force F=20 N is applied to the block. The coefficient of kinetic friction between the block and the plane of rotation is $\mu_k$=0.15.




5% 1- Draw the free body diagram of the 5 kg block showing the forces that act in the plane of motion.

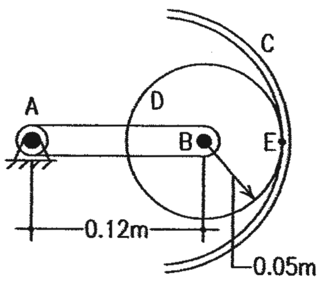
5% 2- Draw the impulse and momentum diagrams of the block between t=0s and t=3s.

10% 3- Determine the velocity of the block at t=3s using the principle of angular impulse and momentum.

5% 4- Determine the distance traveled by the block from t=0s to t=3s using the principle of work and energy and the velocity from part 3 above.

No.2 – (25%)

Arm AB rotates about the fixed axis A with a constant angular velocity of 20 rad/s counterclockwise. The outer disk C is stationary. No slippage occurs between the outer disk C and the inner disk D. For the position shown,




5% 1- Draw the absolute velocity diagram of the mechanism.

5% 2- Determine the velocity of point B using Cartesian form.

5% 3- Locate the instantaneous center of zero velocity of disk D and determine the angular velocity of disk D.

5% 4- Draw the absolute acceleration diagram of the mechanism.

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USE FOR ANSWER

25%

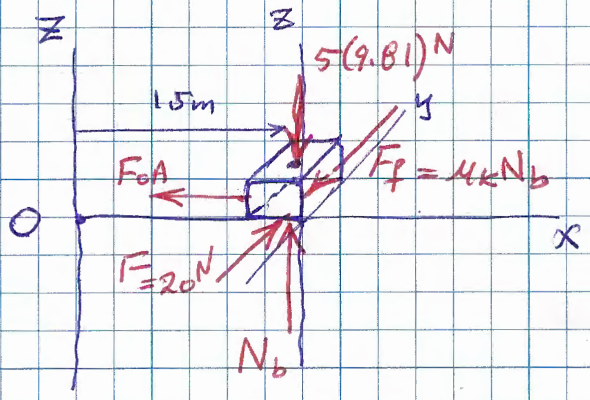
## PROBLEM 1 :

Given $F = 20\,N$ ; $m_b = 5\,kg$ ; $v = 2\,m/s$ ; $\mu_k = 0.15$

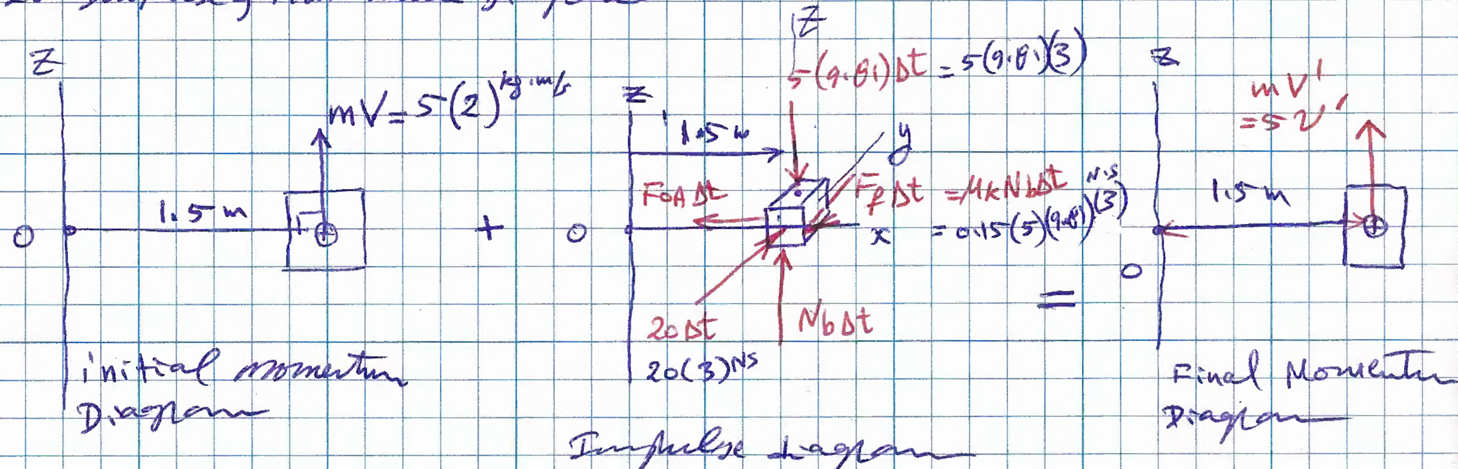
1 – Draw F.B.D in 3-D

$+\uparrow \Sigma F_z = 0$

$N_b = (5)(9.81) = 49.05\,N$




2. Draw Impulse & Momentum Diagrams




3. $\uparrow \Sigma(\text{moments})_{O_z}$ : $1.5^{m}(5)^{kg}(2)^{m/s}\ kg\cdot m^2/s + 20^{N}(3^{s})(1.5^{m}) - \underbrace{0.15(5)(9.81)(3)}_{F_f \cdot \Delta t}(1.5)\ (d_\perp)$

$= (1.5\,m)(5^{kg})(v'^{\,m/s})$

$15\ kg\cdot m^2/s + 90\ N\cdot s\cdot m - 33.10875\ N\cdot s\cdot m = 7.5\,v$

$71.89125\ N\cdot s\cdot m = 7.5\,v$ ; $v = 9.5855\ m/s$ **Ans.**

4. $T_1 + \Sigma U_{1-2} = T_2$ ; $T_1 = \frac{1}{2} m v_1^2 = \frac{1}{2}(5)(2)^2 = 10\,J$

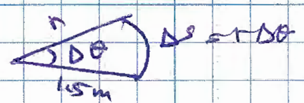
$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2}(5)(9.5855)^2 = 229.7045\,J$

$\Sigma U_{1-2} = 20\,\Delta S - \overset{\mu_k}{0.15}(5)(9.81)\,\Delta S = 20\Delta S - 7.3575\,\Delta S$

$\Sigma U_{1-2} = 12.6425\,\Delta S$

$\therefore\ 10 + 12.6425\,\Delta S = 229.7045$

$\Delta S = \dfrac{219.7045}{12.6425} = 17.378\,m$ ; $\Delta\theta = \dfrac{\Delta S}{r} = \dfrac{17.378}{1.5\,m}$

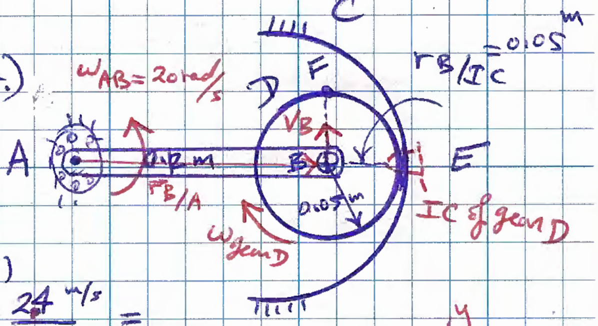



$= 11.585\ rad \,/\, 2\pi\ rad/rev. = 1.844\ rev.$

**PROBLEM 2:** Given $\omega_{AB} = 20\ rad/s$ ↺ $=$ Constant $\Rightarrow \vec{\alpha}_{AB} = \vec{0}$.

1. Draw the absolute velocity diagram of mechanism

2. Find $\vec{V}_B$ using vector analysis



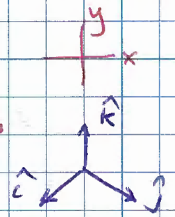

$$\vec{V}_B = \vec{\omega}_{AB} \times \vec{r}_{B/A\,fixed} \quad (\text{Link } AB: \text{Fixed axis rot.})$$

$$\vec{V}_B = 20\hat{k} \times 0.12\hat{i} = 2.4\,\hat{j}\ m/s \quad \text{Ans.}$$

3. Find $\vec{\omega}_D$: Gear D: (general plane motion)

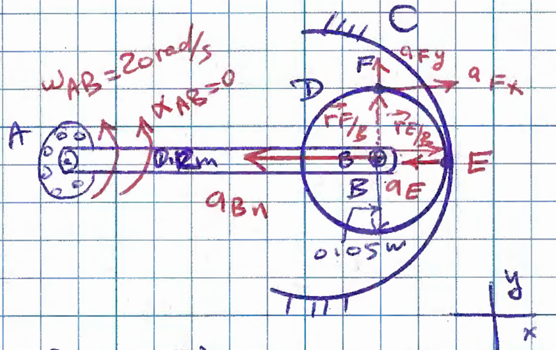
$$V_B = \omega_{gear\,D}\, r_{B/IC}\ ;\ \omega_{gear\,D} = \frac{2.4\ m/s}{0.05\ m} =$$

$$\omega_{gear\,D} = 48\ rad/s\ \circlearrowright \quad \vec{\omega}_{gear\,D} = -48\hat{k}\ rad/s \quad \text{Ans.}$$




4. Draw the absolute acceleration Diagram of the mechanism.

5. Find $\vec{\alpha}_{gear\,D}$.




Link AB: $\omega_{AB} = 20\ rad/s = \text{constant} \Rightarrow \alpha_{AB} = 0$

$$\therefore a_{Bt} = 0\ ;\ a_{Bn} = \omega_{AB}^2\, r_{B/A\,fixed} = 20^2(0.12)$$

$$a_{Bn} = 48\ m/s^2 = a_B$$

Gear D:

$$\vec{a}_E = \vec{a}_B + \vec{\alpha}_{gD} \times \vec{r}_{E/B} - \omega_{gD}^2\, \vec{r}_{E/B}$$

$$-a_E\hat{i} = -48\hat{i} + \alpha_{gD}\hat{k} \times 0.05\hat{i} - (48)^2(0.05\hat{i})$$

$$-a_E\hat{i} = -48\hat{i} + 0.05\,\alpha_{g.D}\hat{j} - 115.2\hat{i}\ ;\ \hat{i}: -a_E = -48 - 115.2$$

$$\hat{j}: 0 = 0.05\,\alpha_{g.D}\ ;\ \alpha_{g.D} = 0.$$

$$a_E = 163.2\ m/s^2 \leftarrow$$

$$\boxed{\vec{a}_E = -163.2\hat{i}\ m/s^2}$$

$$\boxed{\vec{\alpha}_{g.D} = \vec{0}}$$

$$\vec{a}_F = \vec{a}_B + \vec{\alpha}_{g.D} \times \vec{r}_{F/B} - \omega_{gD}^2\, \vec{r}_{F/B}$$

$$a_{Fx}\hat{i} + a_{Fy}\hat{j} = -48\hat{i} + 0 - (48)^2\, 0.05\hat{j}$$

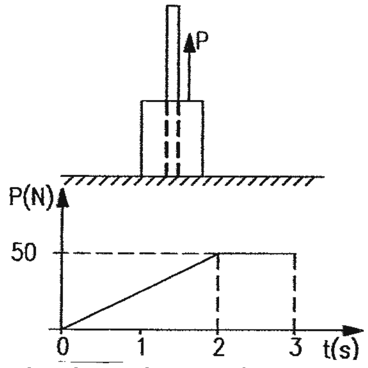
$$= -48\hat{i} - 115.20\hat{j}$$

$$\therefore \boxed{\vec{a}_F = -48\hat{i} - 115.2\hat{j}}\ m/s^2 \quad \text{Ans.}$$

5% 5- Determine the angular acceleration of disk D and the acceleration of point E.

No.3– (25%)

A 2 kg collar which can slide on a frictionless vertical rod is acted upon by a force P which varies in magnitude as shown. The collar is initially at rest.




5% 1- Determine the instant of time when the collar starts moving.

10% 2- Determine the velocity of the collar at t=2s, include the impulse and momentum diagram of the collar.

5% 3- Determine the velocity of the collar at t=3s.

5% 4- Determine the distance travelled by the collar between t=2s and t=3s.

No.4– (25%)

A 1500 kg car is moving up a 7% grade street at a constant speed of 15 m/s.

5% 1- Draw the free body diagram of the car.

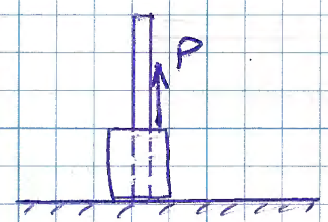
10% 2- Determine the traction force that moves the car.

5% 3- Determine the power that must be delivered to the wheels.

5% 4- Determine the power developed by the engine if the car mechanical efficiency is 0.65.

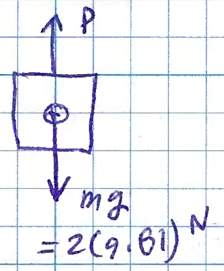
## PROBLEM 3:

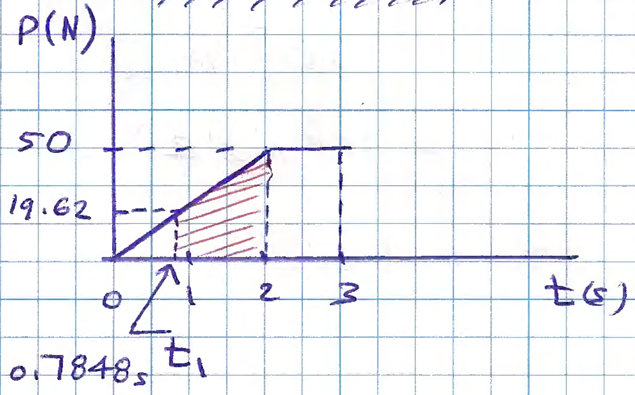
Given $m_{collar} = 2\,kg$ $\quad V_{o\,collar} = 0$ (rest)




1. Find $t$ @ which collar starts moving.

@ equilibrium:




$+\uparrow \Sigma F_y = 0$

$P - 2(9.81) = 0$

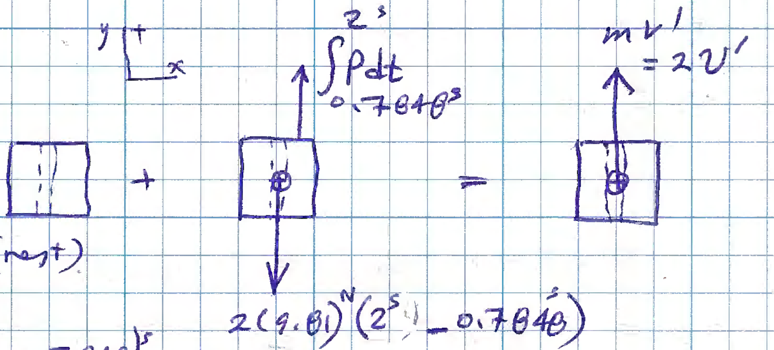
$P = 19.62\,N$

similar Δ's: $\frac{t_1}{2} = \frac{19.62}{50}$ ; $t_1 = 0.7848\,s$

$t_1 = 0.7848\,s$ Ans.

2. Find $V_{collar}$ @ $t = 2\,s$

Draw the I. & M. Diagrams:




$+\uparrow \Sigma(\text{vectors})_y$:

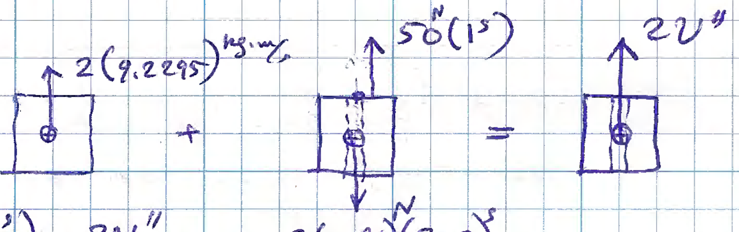
$0 + \int_{t_1 = 0.7848^s}^{2^s} P\,dt - 2(9.81)^N(2 - 0.7848)^s = 2V'$

$\text{Area}\Big|_{0.7848}^{2} - 23.8422^{N \cdot s} = 2V'$ ; $V' = \frac{18.4589}{2} = 9.2295$

$\text{Area} = \frac{(19.62 + 50)(2 - 0.7848)}{2} = 42.3011\,N \cdot s$

$V' = 9.2295\,m/s$ Ans.

3. Find $V_{collar}$ @ $t = 3\,s$




$+\uparrow \Sigma \text{vectors}_y$:

$+\uparrow\ 2(9.2295) + 50(1) - 2(9.81)(1^s) = 2V''$

$48.839 = 2V''$

$V'' = 24.4195\,m/s$ Ans.

4. $T_1 + \Sigma U_{1-2} = T_2$ ; $T_1 = \frac{1}{2}(2)(9.2295)^2 = 85.18367\,J$

$\Sigma U_{1-2} = 50\,\Delta Y - 2(9.81)\Delta Y$ ; $T_2 = \frac{1}{2}(2)(24.4195)^2 = 596.312\,J$

$85.18367 + 50\,\Delta Y - 2(9.81)\Delta Y = 596.312$ ; $\Delta Y = \frac{511.12833}{30.38}$

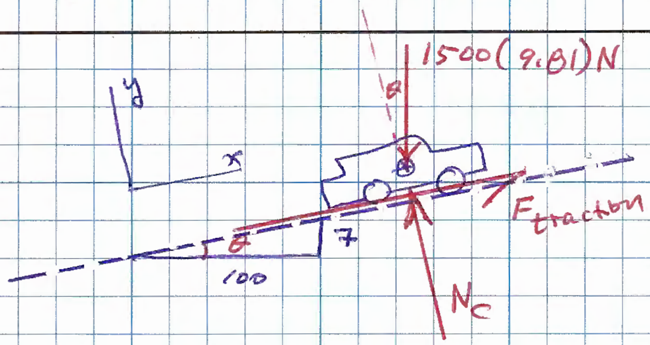
$\Delta Y = 16.8245\,m$ Ans.

PROBLEM 4 :

$m_{car} = 1500\ kg$

$v_{constant} = 15\ m/s$

1. F.B.D of Car




2. Find power (developed by the car engine) ↳ that must be delivered to the wheels

$$P_{output} = F_{traction} \cdot v_{car}$$

$$\nearrow^{+} \Sigma F_x = m a_x \,; \quad F_{traction} - 1500(9.81)(\sin\theta) = 0 \quad \text{(constant vel.)}$$

$$F_{traction} = 1500(9.81)(\sin 4^\circ); \quad \theta = \tan^{-1}\left(\frac{7}{100}\right) = 4^\circ$$

$$= 1026.4665\ N$$

$$\therefore P_{output} = 1026.4665^{N} \cdot \left(15^{m/s}\right) = 15\,396.997\ W$$

$$P_{output} = 15.4\ kW \quad \text{Ans.}$$

$$P_{input} = \frac{15.4}{0.65} = 23\,687.688\ W = 23.7\ kW \quad \text{Ans.}$$